CASE STUDY ANALISIS

Master MMTC 24-25

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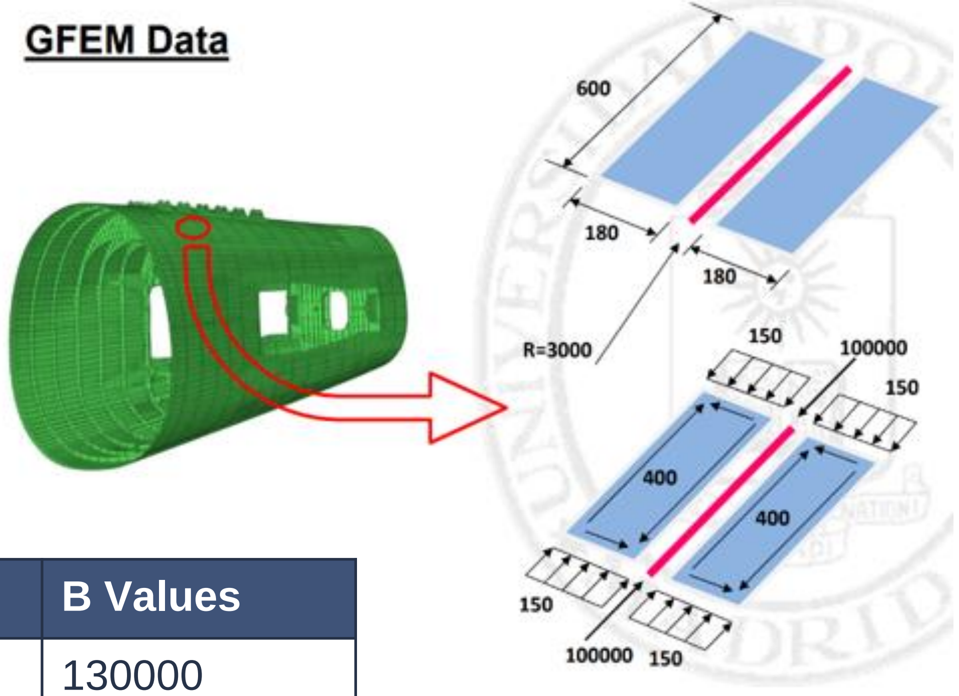
1. PROBLEM DEFINITION

- Unpressurised Rear Fuselage for a large aircraft.
- Structure data:
  - CFRP material
  - AFP lay up
  - Co-bonded skin/stringer construction technology
- Damage Tolerance Allowables:
  - Skin: -4000 $\mu\varepsilon$
  - Stringer: -3200 $\mu\varepsilon$



	Mean Values	B Values
E_{11}	150000	130000
E_{22}	10000	8500
G_{12}	4500	4000
ν_{12}	0.3	0.3
t	0.18	0.18

2. STRINGER DAMAGE TOLERANCE

PRESIZING

Using the estimation of orthotropic laminate properties and a recommended laminate for a stringer (60/30/10),to obtain the values it is used the B values given by the problem

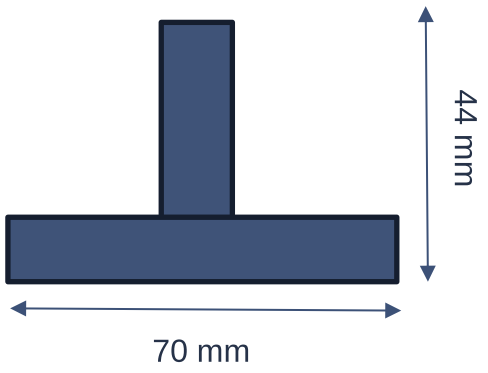
Simplified Rule (Based in % of plies in each direction & E0)		
		%Error
$E_x = \dfrac{(\%0)\left[1-\frac{(\nu-\nu_0)\nu_0}{10}\right]+(\%\pm45)\left[\frac{1-\nu}{4}+\frac{1-\nu\nu_0}{10}\right]+(\%90)\left[\frac{1-\nu\nu_0}{10}\right]}{100}E_0 =$	**88331**	1,5
$E_y = \dfrac{(\%90)\left[1-\frac{(\nu-\nu_0)\nu_0}{10}\right]+(\%\pm45)\left[\frac{1-\nu}{4}+\frac{1-\nu\nu_0}{10}\right]+(\%0)\left[\frac{1-\nu\nu_0}{10}\right]}{100}E_0 =$	**33185**	3,5
$G_{xy} = \dfrac{0.264\,(\%\pm45)+0.028\,(\%0+\%90)}{100}E_0 =$	**11821,33**	-0,5
$\upsilon_{xy} = \dfrac{2.5(\%\pm45)+100\nu_0}{2.5(\%\pm45)+9(\%\,90)+100} =$	**0,34**	7,4

PRESIZING

$$\sigma_{xx} = \frac{F_{xx}}{A} = \varepsilon . E$$

Number of plies	Lay-up	%
19	0	60
8	+/-45	27
4	90	13

- $A = \frac{F_{xx}}{\varepsilon * E} = \frac{100000}{3200 * 10^{-6} * 88331} = 353{,}783\ \mathrm{mm}^2$
- Stacking Sequence: (+45/-45/0/0/0/-45/+45/0/0/0/90/0/90/0/0)S
- Laminate thickness: t = 0,18*30 = 5,4 mm
- Cross- Section: A = $(\frac{w}{2} + h)$*t*nº of plies --> A= 79*0,18*30 = 426,6 mm^2



ANALYSIS

ABD Stiffness Matrix:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x{}^0 \\ \varepsilon_y{}^0 \\ \gamma_{xy}{}^0 \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} 487612 & 56400 & 0 & 0 & 0 & 0 \\ 56400 & 179619 & 0 & 0 & 0 & 0 \\ 0 & 0 & 64148 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1090720 & 225050 & 3564 \\ 0 & 0 & 0 & 225050 & 354654 & 3564 \\ 0 & 0 & 0 & 3564 & 3564 & 243879 \end{bmatrix} \begin{Bmatrix} \varepsilon_x{}^0 \\ \varepsilon_y{}^0 \\ \gamma_{xy}{}^0 \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

Balanced	No shear deformation under tension loads
Symmetrical	No bending or Twisting deformation under in plane loads
Not Specially Orthotropic	Twisting deformation under bending loads

ANALYSIS

Inverse ABD Elastic Matrix:

$$\begin{Bmatrix} \varepsilon_x{}^0 \\ \varepsilon_y{}^0 \\ \gamma_{xy}{}^0 \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \cdot \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \boxed{\begin{matrix} -0.003192 \\ 0.001002 \\ 0.000000 \end{matrix}} \\ 0.000000 \\ 0.000000 \\ 0.000000 \end{Bmatrix} = \begin{bmatrix} 2.128E-06 & -6.682E-07 & 0 & 0 & 0 & 0 \\ -6.682E-07 & 5.777E-06 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.559E-05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.055E-06 & -6.694E-07 & -5.635E-09 \\ 0 & 0 & 0 & -6.694E-07 & 3.245E-06 & -3.764E-08 \\ 0 & 0 & 0 & -5.635E-09 & -3.764E-08 & 4.101E-06 \end{bmatrix} \begin{Bmatrix} \boxed{\begin{matrix} -1500 \\ 0 \\ 0 \end{matrix}} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

ANALYSIS

Damage tolerance allowable:

Stringer: -3200 με

DEFORMATION: 3192 με< 3200 με

The deformations in the laminate proposed for the super stringer do not exceed the maximum allowable.

3. SKIN DAMAGE TOLERANCE

SKIN: LAMINATE

Since there is no defined skin thickness requirement in the specification, we hypothesize that the laminate must be:

1. Symmetrical
2. Balanced
3. Orthotropic
4. Fiber orientation: 0º, ±45º y 90º

➢ Symmetric tape laminates with LSS: $((+45º/-45º/(*)/-45º/+45º/(*)/-45º/+45º/(*)/+45º/-45º/(*))_n)s$
being: (*) Whichever number of 0º and/or 90º plies.

STACKING SEQUENCE: (+45º/-45º/0º/-45º/+45º/0º/-45º/+45º/90º/+45º/-45º/0º)S

Orientation (º)	Number of plies	Percentage (%)
0	6	25
±45º	16	66.67
90	2	8.33
Total number of plies		24

TOTAL SKIN THICKNESS:
24*0.18=4.32 mm

SKIN: ABD STIFFNESS MATRIX

- By entering the chosen laminate into de Excel sheet, the stiffness matrix ABD necessary to calculate the buckling effects is obtained.
- Because it is a symmetric laminate, the terms of matrix B (Membrane-Bending coupling stiffness) are 0.
- $A_{16} = A_{26} = 0 \rightarrow Balanced$ → Elimination of extension-shear coupling
- $D_{16} = D_{26} = 0 \rightarrow Specially\ orthotropic$ → This eliminates the damaging effect of coupling torsion- bending.

Flows	ABD Matrix	Strains

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x{}^0 \\ \varepsilon_y{}^0 \\ \gamma_{xy}{}^0 \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

ABD Matrix

259833	96178	0	0	0	0
96178	171835	0	0	0	0
0	0	102377	0	0	0
0	0	0	405591	165659	0
0	0	0	165659	233573	0
0	0	0	0	0	175299

Balanced	No shear deformation under tension loads
Symmetrical	No bending or Twisting deformation under in plane loads
Specciaaly Orthotropic	No Twisting deformation under bending loads

SKIN: LAMINATE MEAN PROPERTIES

Simplified Rule **(Based in % of plies in each direction & E0)**

		%Error
$E_x = \dfrac{(\%\mathbf{0})\left[1-\frac{(\nu-\nu_0)\nu_0}{10}\right]+(\%\pm\mathbf{45})\left[\frac{1-\nu}{4}+\frac{1-\nu\nu_0}{10}\right]+(\%\mathbf{90})\left[\frac{1-\nu\nu_0}{10}\right]}{100}E_0 =$	**49493**	3.8
$E_y = \dfrac{(\%\mathbf{90})\left[1-\frac{(\nu-\nu_0)\nu_0}{10}\right]+(\%\pm\mathbf{45})\left[\frac{1-\nu}{4}+\frac{1-\nu\nu_0}{10}\right]+(\%\mathbf{0})\left[\frac{1-\nu\nu_0}{10}\right]}{100}E_0 =$	**29798**	-5.5
$G_{xy} = \dfrac{0.264\,(\%\pm\mathbf{45})+0.028\,(\%\mathbf{0}+\%\mathbf{90})}{100}E_0 =$	**24093.3**	1.7
$\upsilon_{xy} = \dfrac{2.5(\%\pm\mathbf{45})+100\nu_0}{2.5(\%\pm\mathbf{45})+9(\%\,\mathbf{90})+100} =$	**0.58**	2.8

SKIN: INVERSE ABD STIFFNESS MATRIX

$$\begin{pmatrix} \bar{N} \\ \bar{M} \end{pmatrix} = \begin{pmatrix} \bar{A} & \bar{B} \\ \bar{B} & \bar{D} \end{pmatrix} \cdot \begin{pmatrix} \bar{\varepsilon} \\ \bar{\kappa} \end{pmatrix} \Longrightarrow \begin{pmatrix} \bar{\varepsilon} \\ \bar{\kappa} \end{pmatrix} = \begin{pmatrix} \bar{A} & \bar{B} \\ \bar{B} & \bar{D} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \bar{N} \\ \bar{M} \end{pmatrix}$$

- INITIAL CONDITIONS:
- ✓ Nx= -150 N/mm.
- ✓ Ny= 0 N/mm
- ✓ Nxy= -400 N/mm

$$\begin{Bmatrix} \boxed{\begin{matrix} -0.000728 \\ 0.000408 \\ -0.003907 \end{matrix}} \\ 0.000000 \\ 0.000000 \\ 0.000000 \end{Bmatrix} = \begin{bmatrix} 4.85E-06 & -2.72E-06 & 0 & 0 & 0 & 0 \\ -2.72E-06 & 7.34E-06 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.77E-06 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.47E-06 & -2.46E-06 & 0 \\ 0 & 0 & 0 & -2.46E-06 & 6.03E-06 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.7E-06 \end{bmatrix} \begin{Bmatrix} \boxed{\begin{matrix} -150 \\ 0 \\ -400 \end{matrix}} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

SKIN: INVERSE ABD STIFFNESS MATRIX

- Skin damage tolerance allowable: -4000 με

- Deformation **εx**: 728 με< 4000 με
- Deformation **εy**: 408 με< 4000 με
- Deformation **Υxy**: 3907 με< 4000 με

4. STRINGER LOCAL BUCKLING

STRINGER LOCAL BUCKLING

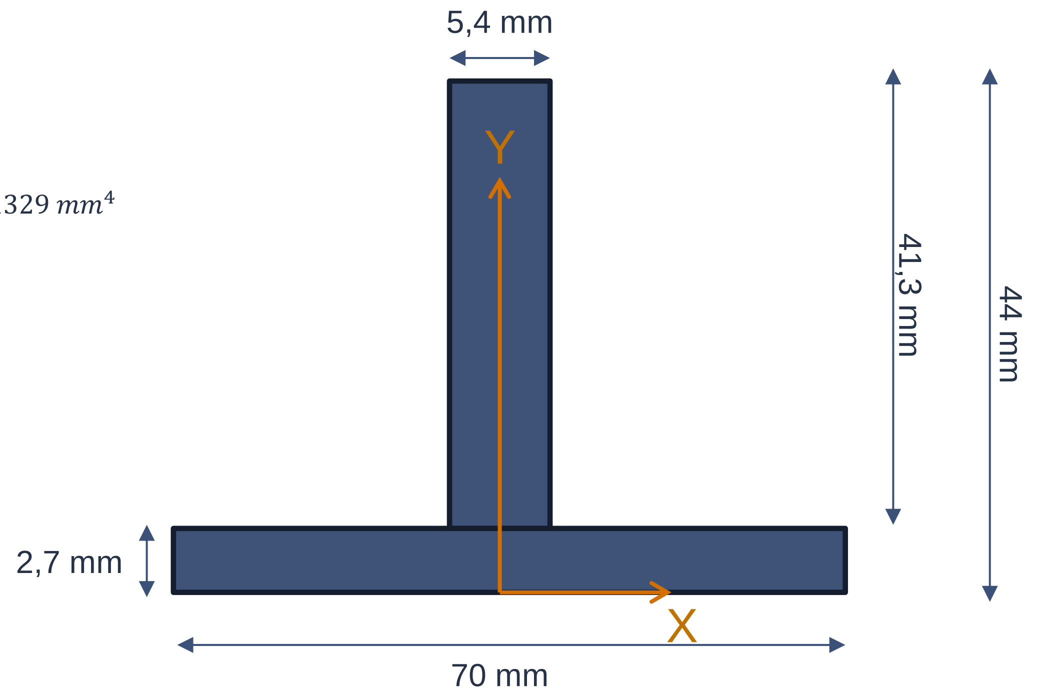
Geometric and Inertial Properties:

- $Area = A_{web} + A_{flange} = 412{,}02\ mm^2$
- $I_{web} = \frac{1}{12} \cdot b_w \cdot h_w^3 = 31700\ mm^4$
- $I_{flange} = \frac{1}{12} \cdot b_f \cdot h_f^3 = 115\ mm^4$

$$I_X = I_{web} + I_{flange} + A_w \cdot (Y_w - Y_{CG})^2 + A_f \cdot \left(Y_f - Y_{CG}\right)^2 = 81329\ mm^4$$

- $Y_{CG} = \frac{A_w \cdot Y_w + A_f \cdot Y_f}{A_w + A_f} = 13{,}26\ mm$
- $I_{web} = \frac{1}{12} \cdot h_w \cdot b_w^3 = 542\ mm^4$
- $I_{flange} = \frac{1}{12} \cdot h_f \cdot b_f^3 = 77175\ mm^4$

$$I_Y = I_{web} + I_{flange} = 77717\ mm^4$$



	b	h	A	CG
Web	5,4	41,3	223,02	(0 ; 23,35)
Flange	70	2,7	189	(0 ; 1,35)

STRINGER LOCAL BUCKLING

$$P_c = \frac{\pi^2 \cdot E \cdot I}{(KL)^2} = 188202\ N$$

- P_c = Critical Load to Induce Buckling
- E = Elastic Modulus
- I = Minimum Moment of Inertia
- L = Unsupported Length of the Column
- K = Column Effective Length Factor

$$\boxed{RF = 1{,}25}$$

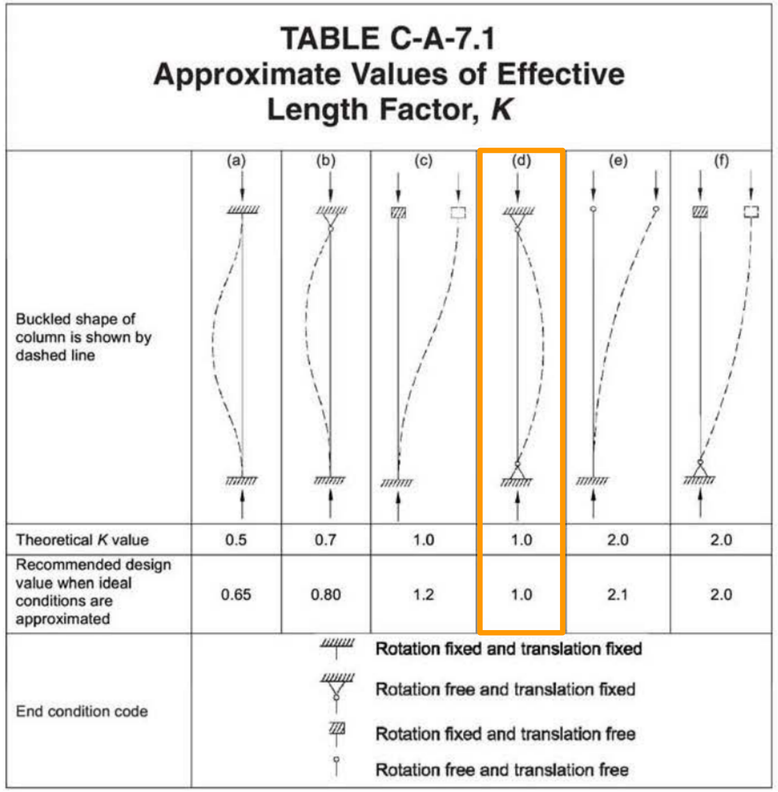


TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0

End condition code:
- Rotation fixed and translation fixed
- Rotation free and translation fixed
- Rotation fixed and translation free
- Rotation free and translation free

5. SKIN LOCAL BUCKLING

Skin Local Buckling: orthotropic panel in compression

$$Z = \frac{b^2}{r*t} = \frac{180^2}{3000*4.32} = 2.5 < 5$$

(effect of curvature assumed negligible)

$$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} = \begin{bmatrix} 405591 & 165659 & 0 \\ 165659 & 233573 & 0 \\ 0 & 0 & 175299 \end{bmatrix}$$

$$F_{xxb} = -\frac{2\pi^2}{b^2}\left(\sqrt{D_{11}D_{22}} + D_{12} + 2D_{33}\right)$$

$$\varepsilon_{xxb} = \frac{F_{xxb}}{tE_c}$$

$$F_{xxb} = -\frac{2\pi^2}{180^2}\left(\sqrt{405591 * 165659} + 233573 + 2 * 175299\right) = -502.04\ \mathrm{N/mm}$$

$$\varepsilon_{xxb} = \frac{502.04}{4.32*49493} = -0.002348$$

$$RF_{\varepsilon_{xxb}} = \frac{\varepsilon_{xxb}}{\boxed{\varepsilon_{xx}}} = \frac{-0.002348}{-0.000728} = 3.23$$

$$\varepsilon_{xx} = N_{xx} * A^{*}{}_{11} = -0.000728$$

Skin Local Buckling: orthotropic panel in shear

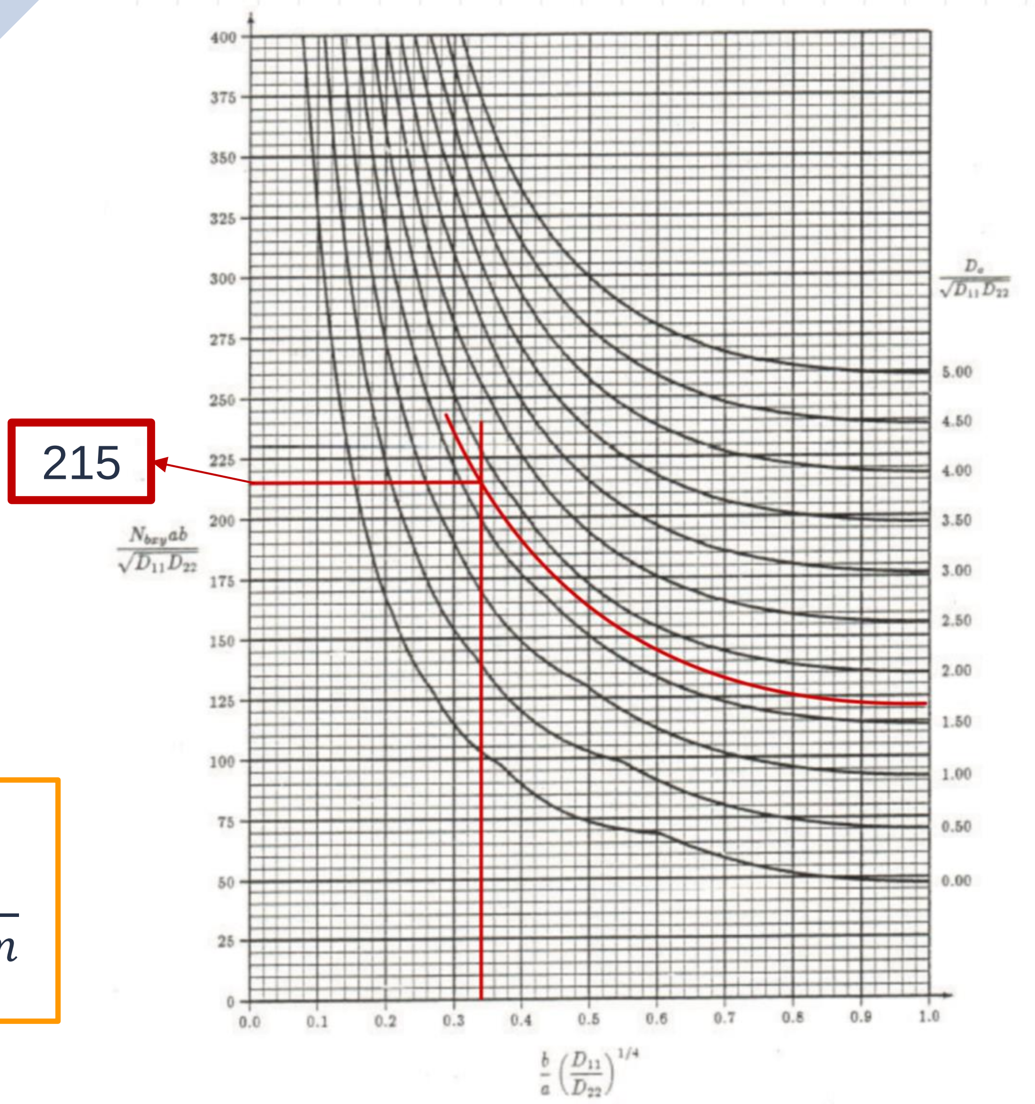
$$D_0 = D_{12} + 2D_{33} = 165659 + 2 * 175299 = 516257$$

$$\frac{D_0}{\sqrt{D_{11}D_{22}}} = \frac{516257}{\sqrt{405591 * 233573}} = 1.68$$

$$\frac{b}{a}\left(\frac{D_{11}}{D_{22}}\right)^{1/4} = \frac{180}{600}\left(\frac{405591}{233573}\right)^{1/4} = 0.34$$

$$\frac{N_{bxy}ab}{\sqrt{D_{11}D_{22}}} = 215$$

$$N_{bxy} = \frac{215 * \sqrt{D_{11}D_{22}}}{ab} = \frac{215 * \sqrt{405591 * 233573}}{600 * 180} = 612.73\,\frac{N}{mm}$$



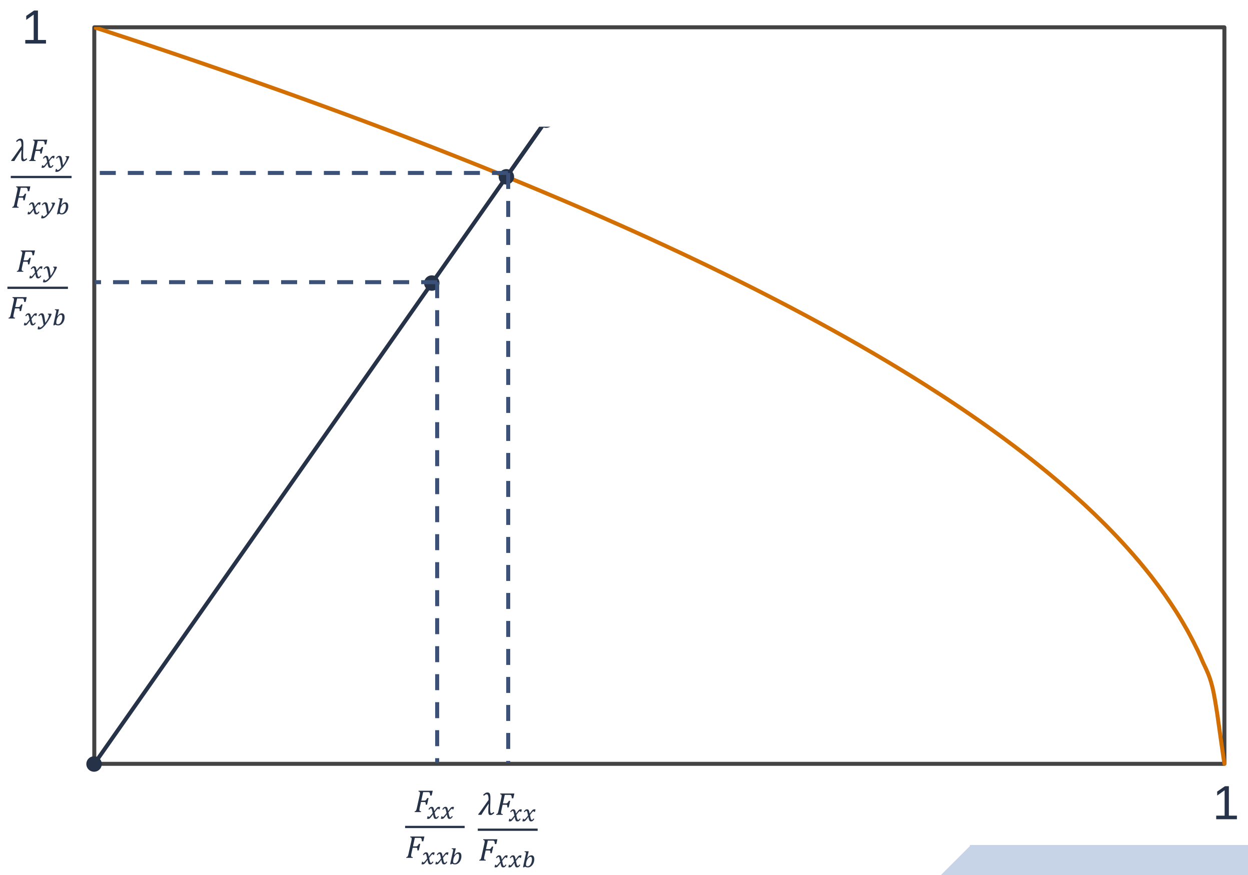
Skin Local Buckling: orthotropic panel in compression + shear

$$\frac{F_{xx}}{F_{xxb}} = \frac{-150}{-502.04} = 0.299$$

$$\frac{F_{xy}}{F_{xyb}} = \frac{400}{612.73} = 0.653$$

$$\frac{\lambda F_{xx}}{F_{xxb}} + \left(\frac{\lambda F_{xy}}{F_{xyb}}\right)^2 = 1 \qquad \lambda = 1.221$$

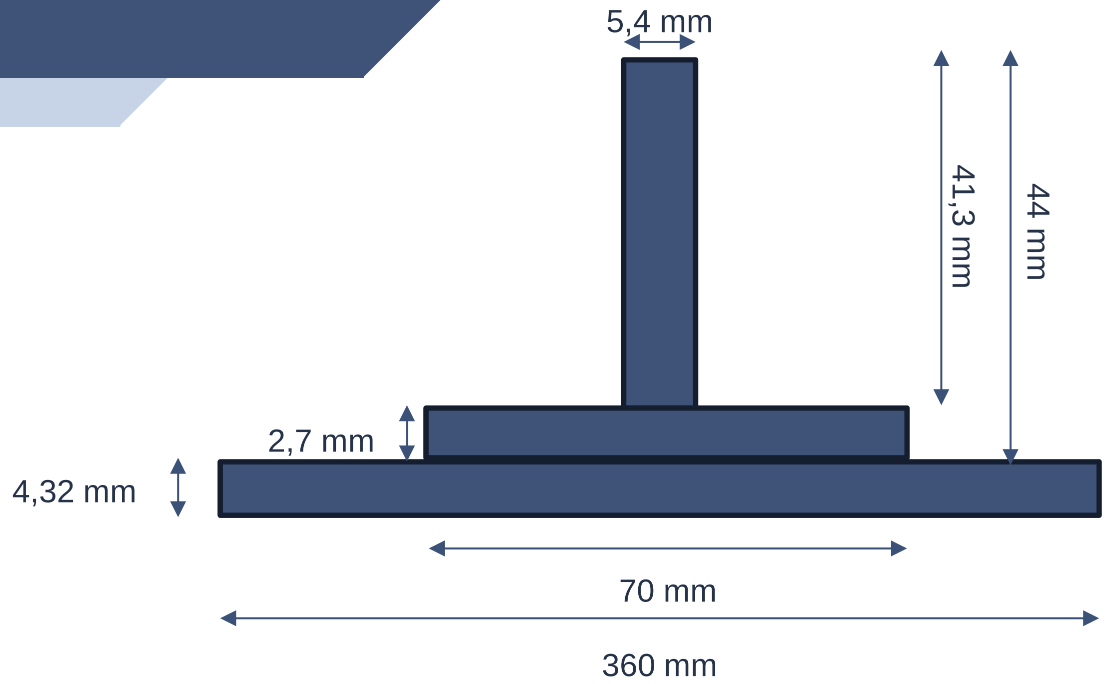
$$RF_{xx+xy} = 1.221$$



6. GLOBAL BUCKLING

GLOBAL BUCKLING

INERTIA		A (mm^2)	I (mm^4)	Zcg (mm)
skin h	4,32	1555,2	2418,647	2,16
skin w	360			
foot h	2,7	189	114,8175	5,67
foot w	70			
web h	41,3	223,02	31700,25	27,67
web w	5,4			
Total		1967,22	34233,71	5,38



$$P_{cb} = -\frac{\pi^2 E_{cB} I}{L^2} \cdot \frac{1}{1 + \frac{E_{cM} I}{G_M A} \cdot \frac{\alpha \pi^2}{L^2}}$$

$$\alpha = \frac{SA}{It}$$

$$G^{eq}{}_{xy} = \frac{1}{A^*{}_{66} \cdot t}$$

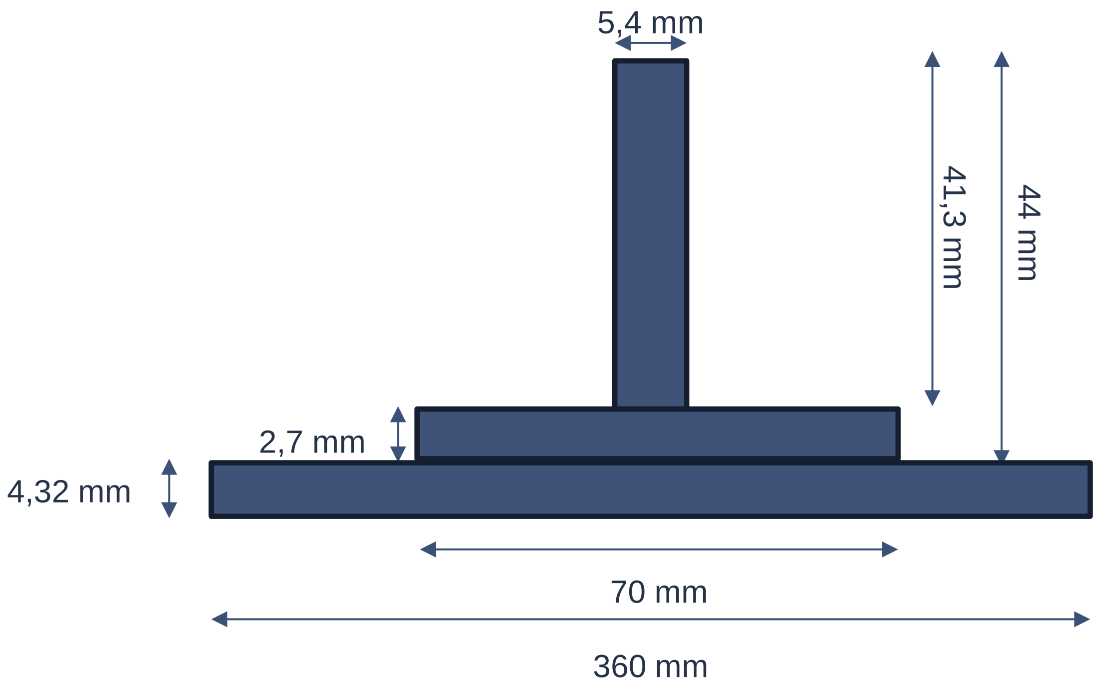
$$E^{eq}{}_{x} = \frac{1}{A^*{}_{11} \cdot t}$$

GLOBAL BUCKLING	
I_y	161180,5
S	5022,11
Ecm	45576,9
Ecb	40036,64
Gm	21898,97

Pcb	168001,1 N
P	154000 N
RF	1,09

CONCLUSION

BUCKLING	RF
Skin Local Buckling (Compression+Shear)	1,221
Stringer Local Buckling	1,25
Global Buckling	1,09



Thank you for your attention!